ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

MODIFICATION OF ENSKOG THEORY FOR THE COMPUTATION OF TRANSPORT CHARACTERISTICS OF REAL GASES AND LIQUIDS

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UDC 536.71

The authors propose to replace the effective potential describing the interaction of real molecules by the potential of solid spheres with diameter depending on the temperature and density. This permits one to extend Enskog theory and its equations of the transport characteristics for describing experimental results on the transport coefficients of real gases.

The dependence of the diameter of the solid spheres on the temperature is logarithmic and follows rigorously from the representation of the repulsion potential in Born–Meyer form,

$$U(r) = \varepsilon \exp(-\alpha r/R_e). \qquad (1)$$

Then

$$\sigma(T) = \frac{R_e}{\alpha} \ln(\varepsilon/LkT). \qquad (2)$$

The presence of the dependence $\sigma(\rho)$ is mainly a manifestation of the effect of the energy of mutual attraction of the molecules and also a consequence of the nonadditive multiparticle interaction energy. The dependence of σ on the density is obtained from experimental data from the condition that this relationship holds.

The possibility of a generalized representation of the collision diameter of different gases using the parameters of the curve of an ideal gas is demonstrated.

The method is illustrated taking the example of the viscosity of Ne, Ar, Xe, and Kr in gaseous and liquid states. The dependence $\sigma^*(\tau, \omega)$ is obtained from the results, where $\sigma^* = \sigma/\mathrm{Re}$, $\Theta = T/T_B$, $\omega = \rho/\rho_0$, Re is the coordinate of the minimum effective potential equal to $\sim \rho_0^{-1/3}$, T_B is Boyle temperature, ρ_0 is the density on the ideal gas curve at T = 0;

$$\sigma^* = \Sigma a_i \omega^i - \Sigma b_i \omega^i \lg \theta. \qquad (3)$$

The coordinates of the ideal gas curve are

	T_B, K	ρ_0	R_e
Ne	122,1	1,673	3,063
Ar	407,76	1,870	3,688
Kr	567,5	3,210	3,9415
Xe	791,0	3,890	4,2935

The data on the viscosity at atmospheric pressure and Eq. (3) enable one to compute the viscosity from Enskog equation written in the form

$$\eta/\eta_0 = \frac{1}{g(\sigma)}\left[1+0.8\left(\frac{b}{v}\right)g(\sigma) + 0.761\left(\frac{b}{v}\right)^2 g(\sigma)^2\right]. \qquad (4)$$

*All-Union Institute of Scientific and Technical Information.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 6, pp. 1107–1122, December, 1975.

The value of g(σ), the radial distribution function of the solid particles at a distance σ, is obtained from the latest data obtained by the method of molecular dynamics.

A detailed comparison of the results with the experimental data on the viscosity of the four inert gases is given. The comparison shows that the error of the computed quantities mainly lies within the ranges of error in the experiment.

Dep. 1773-75, April 9, 1975.
Original article submitted August 6, 1974.

COMPUTATION OF COEFFICIENTS OF THERMAL AND ELECTRICAL CONDUCTIVITIES OF CERTAIN TYPES OF HYPERCONDUCTORS IN THE RANGE 4-300°K

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UDC 536.21

For the computation the entire temperature range is divided into two subranges: the first is 4.2-78°K and the second is 78-300°K. We write the formula for the computation of the coefficient of thermal conductivity in the range 4.2-78°K in the form [1, 2]

$$\lambda(T)=\frac{T}{A+BT^3};\quad A=\rho_0/L. \tag{1}$$

The theoretical value of the Lorentz constant $L = 2.44\cdot10^{-8}\ \Omega\cdot W/K^2$.

For hyperconductors with a copper base $B = 2.5\cdot10^{-7}$ and for hyperconductors with an aluminum base $B = 3.2\cdot10^{-7}$ M/W·K. The residual resistance ρ_0 for alloys from copper B3, M1, M0 and from aluminum A999, A995 are, respectively, equal to 10^{-11}, $3.4\cdot10^{-10}$, $6.4\cdot10^{-10}$, $4.0\cdot10^{-12}$, and $4.0\cdot10^{-11}\ \Omega\cdot m$. Furthermore, the function λ (T) was computed for hyperconductors in the form of an alloy (composition) on copper and aluminum bases with $\rho_0 = 10^{-9}$ and $10^{-8}\ \Omega\cdot m$. For the range 4-78°K the formulas for computing the mean and maximum values λ_m and λ_M (T_i) are of great interest:

$$\lambda\ =0.528/\sqrt[3]{A^2B};\quad T\ =\sqrt[3]{A/2B}\ ; \tag{2}$$

$$\lambda_m=\frac{1}{(T_2-T_1)B}\left\{\frac{1}{6C}\ln\frac{(T/C)^2-(T/C)+1}{[(T/C)+1]^2}+\frac{1}{\sqrt{3}\,C}\operatorname{arc\,tg}\frac{(T/C)-0.5}{\sqrt{3}/2}\right\}\Bigg|_{T=T_1}^{T=T_2},\quad C=\sqrt[3]{A/B}. \tag{3}$$

The specific electrical resistance is computed from Matissen's rule, breaking it into the ideal and the residual components:

$$\rho(T)=\rho_0+\rho_i(T). \tag{4}$$

A numerical analysis of the dependence

$$\rho_i(T)=B_kT^k;\quad k=1,\ 2,\ 3,\ 4, \tag{5}$$

with the use of a computer showed that the cubic dependence (k = 3) gives the best approximating properties in the range 4.2-78°K for hyperconductors on copper, aluminum, nickel, silver bases, when $B_3 = 0.3792\cdot10^{-14}$, $0.4415\cdot10^{-14}$, $0.9816\cdot10^{-14}$, $0.5852\cdot10^{-14}\ \Omega\cdot m/K^3$, respectively. For the range 78-300°K the dependence is linear:

$$\rho_i(T)=\rho_i(78)+B_1(T-78). \tag{6}$$

For hyperconductors on copper and aluminum bases $\rho_i(78) = 0.18 \cdot 10^{-8}$ and $0.21 \cdot 10^{-8}$ $\Omega \cdot m$ and $B_1 = 0.7059 \cdot 10^{-10}$ and $0.1180 \cdot 10^{-9}$ $\Omega \cdot m/K$, respectively.

For the computation of $\lambda(T)$ in the range 78-300°K the following formulas are applicable:

$$\lambda(T) = \lambda_0 [1 + \beta_\lambda (T - T_0)], \quad T_0 = 189\,^\circ K; \tag{7}$$

$$\lambda_0 = \frac{LT_0}{\rho(T_0)}, \quad \beta_\lambda = \frac{1}{T_0} - \frac{B_1}{\rho(T_0)}; \tag{8}$$

$$\rho(T_0) = \rho_0 + \rho_i(78) + B_1 (T_0 - 78); \tag{9}$$

$$\lambda_m = \lambda_0 \left[1 + \beta_\lambda \left(\frac{T_1 + T_2}{2} - T_0\right)\right]. \tag{10}$$

The maximum error in the computation of $\lambda(T)$ from formulas (1)-(10) relative to the experimental or tabulated data does not exceed 20%, while for $\rho(T)$ it does not exceed 7%.

LITERATURE CITED

1. A. Miessner, Thermal Conductivity of Solids, Liquids, Gases, and Their Compositions [Russian translation], Mir, Moscow (1968).
2. A. I. Shal'nikov (editor), Low-Temperature Physics [Russian translation], IL, Moscow (1959).

Dep. 1774-75, April 21, 1975.
Original article submitted December 24, 1975.

TEMPERATURE DISTRIBUTION AND HEAT FLUXES IN AN INSTALLATION FOR OBTAINING MONOCRYSTALS BY THE METHOD OF VERTICAL DIRECTIONAL CRYSTALLIZATION

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UDC 536.24

Optical monocrystals are obtained in a cylindrical container as a result of the crystallization of a melt which proceeds upward from the bottom owing to the drawing off of heat through a rod as the rod and container descend and emerge from the active zone of the heater. It is expedient to divide the entire cycle of obtaining a monocrystal into three periods: the first is from the time of descent of the rod to the start of crystallization; the second is from the start of crystallization to the emergence of the crystal from the heater; the third is when part of the crystal is located inside the heater and the other part is outside it. A diagram of the process for the second period is presented in Fig. 1. For a study of the thermal processes in the installation in the process of descent of the container we measured the temperature at three points of the rod (T_1, T_2, T_3) and we measured the heater temperature and the electric power supplied. The result of the measurement of temperature T_1 showed that crystallization begins at a distance of 30-40 mm from the lower edge of the heater when the power of the latter is on the order of 8.5 kW.

The analysis of the data of numerous experiments showed that the temperature field in the rod does not depend on its rate of descent when the latter varies from 1.6 to 25.4 mm/h and is determined only by the position of the point in space. This made it possible to use the solution of the steady-state heat-conduction problem for the analysis of the temperature field.

The heat-conduction equation for the part of the rod located outside the heater, with allowance for heat transfer by radiation and convection into the surrounding medium and for radiant heat exchange with the end of the heater, was written in the one-dimensional version as follows:

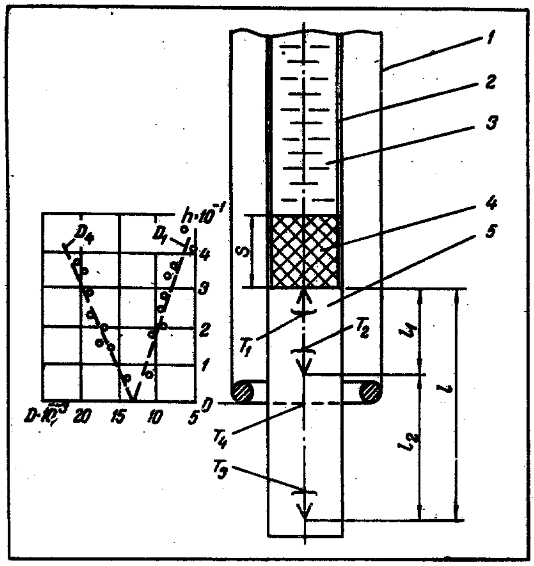

Fig. 1. Diagram for second period of obtaining a monocrystal. $D \cdot 10^{-3}$, deg/m: 1) heater; 2) container; 3) melt; 4) crystal; 5) rod.

$$\frac{d^2T}{dz^2} = \frac{u\sigma_0\varepsilon}{\lambda f} T^4 \Big[1 - \varphi(z) \Big] + \varphi(z) \frac{u\sigma_0\varepsilon_{re}}{\lambda f} (T^4 - T_e^4) + \frac{\alpha u}{\lambda f} (T - T_{av}). \tag{1}$$

If for the part of the rod located inside the heater one takes $\varphi(z) = 1$ and neglects heat transfer through convection, then Eq. (1) can be reduced to the form

$$\frac{d^2T}{dz^2} = BT^4 - A. \tag{2}$$

Since the temperatures at two points of the rod are known from the experiment, by using these data as the boundary conditions in the solution of Eqs. (1) and (2) one can determine the axial temperature gradients and the heat fluxes from the lateral surface of the rod. In this case Eq. (1) was solved numerically on a computer and Eq. (2) was solved approximately using a power series. The results of the calculation of the temperature gradients in the rod at the upper end (D_1) and at the exit cross section of the heater (D_4) are presented in Fig. 1. If for the second period of the process one determines the height s of the crystal and one has data on the temperatures in the installation, then from the solution of the heat-conduction problem one can determine the effective coefficient of thermal conductivity λ_{ef} of the crystal material. The temperature and heat flux will be given at the lower face of the crystal and the crystallization temperature T_L at the upper face.

The extraneous boundary condition allows one to find λ_{ef} from the solution of the inverse problem.

The data obtained are used to create a method for calculating installations designed for obtaining optical monocrystals.

NOTATION

T, temperature; x, z, spatial coordinate; $\varphi(z)$, angular coefficient of an elementary area of the rod relative to the end of the heater; α, heat-transfer coefficient; ε, emissivity; u, perimeter; λ, coefficient of thermal conductivity; f, area; d, diameter; s, length of crystal; ε_{re}, reduced coefficient of radiant exchange.

Dep. 1771-75, April 28, 1975.
Original article submitted December 27, 1974.

EVACUATION AND HEAT-CONDUCTION PROCESSES OF VACUUM SHIELD HEAT INSULATION

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UDC 536.021

An analysis is made of the processes of variation in the pressure of residual gases between layers of vacuum shield heat insulation (VSHI) during its evacuation as a function of the type of perforation (percentage of perforated area, diameter and arrangement of openings).

The experimental study of the residual gas pressure distribution in layers of VSHI and its thermal conductivity coefficient was performed in a pressure chamber on a cylindrical calorimeter and a spherical vessel which were filled with liquid nitrogen during the experiments.

Polyethylene terephthalate film 5 μ thick with two-sided aluminum metallization doubled with glass film served as the starting material for the test specimens. One hundred layers of insulation were wound on the cylindrical calorimeter with a stacking density of 14-15 shields/cm. We tested specimens with shield perforation areas of 1.4% (openings 8 mm in diameter, spacing 60 mm) and 3.14% (openings 2 mm in diameter, spacing 10 mm).

Helium and nitrogen were used as the supercharge gas for the preliminary treatment of the specimen.

The residual gas pressure in the insulation during its evacuation was measured with LT-2 thermocouple manometers and LM-2 ionization manometers through long glass tubes (5×1 mm in cross section, 650 mm long) which were hermetically inserted through the cover of the pressure chamber and placed between layers of insulation.

The temperature distribution in the specimen was determined with copper–Constantan thermocouples 0.1 mm in diameter placed in the layers of insulation.

The residual gas pressure distribution in the insulation and the shield temperature as functions of the duration of the evacuation were constructed from the results of the experiments, and the thermal conductivity coefficient of the VSHI was also calculated.

The experiments performed showed that the type of perforation of the shields has an important effect on the duration and depth of the evacuation of the insulation. The depth of evacuation increases with an increase in the percentage of perforation area of the shields while the evacuation time of the VSHI layers increases with a decrease in perforation. An increase in the percentage of perforation of the shields (from 1.4 to 3.14%), preliminary evacuation, and purging of the insulation with helium and nitrogen reduce the time of evacuation of the VSHI and the time of arrival at a steady state.

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*Deceased.

Dep. 1766-75, December 11, 1974.
Original article submitted February 26, 1973.

MOISTURE TRANSPORT IN THAWING CLAY SOILS

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UDC 551.343.7

If the thawing of clay soils is accompanied by the evaporation of moisture at the surface then a redistribution of moisture occurs in the thawed zone.

Let us consider the simplest case of moisture transport during the thawing of a flat wall, assuming that the total moisture in the frozen soil (u_a) is uniformly distributed, at the surface of the soil the moisture u_0 = const ($u_0 < u_a$), and the moisture transport occurs under isothermal conditions with a known law of motion of

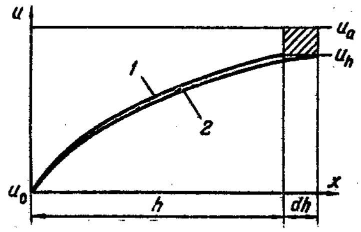

Fig. 1. For derivation of the boundary condition at the moving boundary.

the thawing boundary. At the moving boundary of thawed soil the moisture u_h will always be somewhat less than u_a, i.e., a jump in moisture will be observed at the thawing boundary [1].

At the time τ (Fig. 1) the depth of thawing will be h, with the moisture in the thawed zone being distributed according to some curve 1. In a time $d\tau$ the soil thaws by an amount dh and moisture will be drawn off from this layer into the thawed zone in an amount $\gamma_0(u_a - u_h)dh$ (shaded part of Fig. 1) owing to the moisture gradient, while the moisture is described by curve 2. Consequently, we are correct to write

$$\gamma_0 a_m(u) \left.\frac{\partial u_{(x,\tau)}}{\partial x}\right|_{x=h} \cdot d\tau = \gamma_0 (u_a - u_h)\, dh, \tag{1}$$

where γ_0 is the volumetric mass of the soil skeleton.

Equation (1) is also a condition at the moving boundary.

Taking the coefficient of potential conduction a_m = const and the motion of the thawing boundary in accordance with the law $h = \beta\sqrt{\tau}$ and solving the well-known differential equation of mass conduction [2], we find the moisture distribution in the thawed zone at any time.

In the presence of moisture evaporation at the surface the solution has the form

$$u_{(x,\tau)} = u_0 + \frac{\mu\sqrt{\pi}\,(u_a - u_0)\exp\mu^2}{1 + \mu\sqrt{\pi}\operatorname{erf}\mu\exp\mu^2} \operatorname{erf}\frac{x}{2\sqrt{a_m\tau}},$$

while in the presence of wetting of the soil surface or condensation of the moisture, when $u_0 > u_a$,

$$u_{(x,\tau)} = u_0 - \frac{\mu\sqrt{\pi}\,(u_0 - u_a)\exp\mu^2}{1 + \mu\sqrt{\pi}\operatorname{erf}\mu\exp\mu^2} \operatorname{erf}\frac{x}{2\sqrt{a_m\tau}},$$

where $\mu = \beta/2\sqrt{a_m}$.

Obviously, such a problem can also be solved with other boundary conditions at the stationary boundary, but the condition (1) must be observed at the moving boundary for any law of its motion.

LITERATURE CITED

1. F. Ya. Novikov, "Moisture conditions of clay soils in a thawing zone," in: Studies of Frozen Ground and Construction Problems [in Russian], Part 4, Komi Knizhnoe Izd. (1971).
2. A. V. Lykov, Transport Effects in Capillary-Porous Media [in Russian], Gos. Izd. Tekh. i Teor. Lit., Moscow (1954).

Dep. 1768-75, April 14, 1975.
Original article submitted September 10, 1974.

NONSTEADY THERMAL CONDITIONS OF A PIPELINE LAID IN THAWING SOIL

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UDC 518.517.9:536.2

The problem of the thermal interaction of a pipeline with soil includes two groups of equations, one of which describes the flow of the product along the pipe (hydrodynamic equation [1]), while the other describes the process of heat conduction for the wall and the soil (nonlinear boundary problem of heat conduction with discontinuity coefficients and with moving boundaries – a problem of the Stefan type [2, 3]). There are also conditions of joining of the unknown functions at the boundaries. Heat exchange between the product and the soil and between the soil and the air proceeds according to Newton's law.

The heat flux at the ends of the region is taken as equal to zero. We further assume that the product moves along a long horizontal pipeline of constant cross section with a given constant velocity (with given temperatures at the starting time and at the pipe entrance), the hydrodynamic equations are transformed into one first-order equation for the temperature of the product, and a formula is obtained giving the pressure distribution of the product along the length of the pipe.

The equation obtained is solved with the help of a Laplace transformation and the properties of the delta function and a formula is found in explicit form for the product temperature distribution through the soil temperature, from which, in particular, the well-known Shukhov formula [1, 6] follows. We note that all the known formulae (and the Shukhov formula in particular) for the product temperature were established earlier only for steady flows of the product and constancy of the initial data. The formula obtained in the work is derived with nonsteady flow of the product, variability of the initial data, and with allowance for phase transitions in the soil.

The solution of the Stefan problem for the soil by the method of [3, 4, 5] comes down to the solution of nonlinear algebraic equations which are solved by the trial-run method with iterations.

As an illustration of the developed method a series of calculations are performed on a computer in the case of the transportation of "hot" oil along an oil pipeline when the parameters of the pipeline vary along its length and with time. From an analysis of the results obtained it follows that after a time interval on the order of 120-200 h the oil pipeline enters into an (arbitrarily) steady state. Results of the calculations are presented in the case of an oil pipeline without heat insulation and with heat insulation whose thickness varies along the length of the pipeline.

LITERATURE CITED

1. I. A. Charnyi, Underground Hydrodynamics and Gas Dynamics [in Russian], Gostekhizdat (1951).
2. M. Yunusov, "Solution of one optimum problem of the distribution of a product moving along a pipeline," Dokl. Akad. Nauk Tadzhik SSR, No. 2, 11 (1974).
3. M. Yunusov, "Solution of one optimum problem of the Stefan type," Zh. Vychisl. Mat. Mat. Fiz., 15, No. 2, 345 (1975).
4. B. M. Budak, A. B. Uspenskii, and E. B. Solov'eva, "Difference method of smoothing the coefficients for the solution of the Stefan problem," Zh. Vychisl. Mat. Mat. Fiz., No. 5, 828 (1965).
5. A. A. Samarskii, Introduction to the Theory of Difference Systems [in Russian], Nauka, Moscow (1971).
6. V. V. Anisimov and M. I. Krinitsyn, Construction of Main Pipelines in Regions of Permafrost [in Russian], Gostoptekhizdat, Moscow (1963).

Dep. 1761-75, May 11, 1975.
Original article submitted September 16, 1974.

HEAT EXCHANGE BETWEEN VENTILATING AIR AND SURROUNDING FROZEN ROCK

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UDC 622.536.24

The temperature conditions of underground mines in frozen rock are of the greatest importance for the stability of the mines and for the maintenance of comfortable working conditions in them. When mines are ventilated with warm air a thawed region develops around them (whose strength and thermophysical properties differ sharply from the properties of frozen rock), as a result of which the mine loses stability.

For a calculation of the temperature conditions of an underground mine located in frozen rock it is necessary to solve the conjugate problem of heat exchange between the ventilating air and the surrounding frozen rock. The problem of heat exchange in rock, which in the general case is a three-dimensional Stefan problem with boundary conditions which vary with time and initial conditions which vary in space, is of the greatest difficulty. The solution of this problem in the complete formulation requires the compiling of complicated programs whose execution occupies a lot of time even on modern high-speed computers.

An approximate method allowing one to calculate the temperatures of the ventilating air and the surrounding rock as well as the configuration and intensity of propagation of the melting halo is developed in the present report. For this purpose in each plane cross section the connection between the air temperature in this section, the heat flux at the wall of the mine, and the position of the melting front is sought using the integral method. Using this connection for the solution of the energy equation and assuming in the first approximation that the melting halo has the shape of a truncated right cone we obtain analytical expressions for the temperature of the ventilating air and the dimensions of the melting halo. The shape of the melting halo can be refined as necessary using the air temperature distribution found. The calculations performed showed that there is no need for this refinement as a rule and the shape of the melting halo obtained in this case differs little from a right cone. The accuracy of the approximate method developed in this article is estimated by a comparison with the known exact solutions.

Dep. 1764-75, May 20, 1975.
Original article submitted October 28, 1974.

EFFECT OF GEOMETRICAL PARAMETERS ON THE ENERGETIC SEPARATION OF STEAM IN A VORTEX TUBE

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UDC 621.181.8

One of the peculiarities of the operation of a steam vortex tube is the considerable magnitude of the differential Joule–Thomson effect in the investigated range of temperatures and pressures, as well as the constancy of the wet steam temperature for different degrees of dryness of the steam. In fact, if one turns to the T vs S diagram for steam one sees that the vapor temperature does not vary in an isobaric process between the limiting curves bounding the wet vapor region. Therefore, the temperature characteristics of a vortex tube operating on wet steam have a different form from those on superheated steam or gases, namely, the temperature characteristic $\Delta t_x = f(\mu)$ has a constant value for a considerable distance along the abscissa if the vapor of the cold stream remains wet and its pressure does not change. In this case when the vapor of the cold stream becomes superheated because of its considerable throttling, the characteristic $\Delta t_x = f(\mu)$ takes on the form of the usual curve, as when the vortex tube operates on a gas. The temperature of the cold stream (when the vapor of the cold stream is wet) is determined from the i vs S diagram of state of the steam, and, of course, the temperature reduction at the cold end of the tube does not depend on changes in such geometrical parameters as the diameter of the diaphragm opening, the nozzle area, the length and diffuser angle of the hot end, and the diameter of the vortex tube, its setup, or other geometrical elements. The same thing can be said about the vapor of a "hot" stream when it remains wet, i.e., its temperature can be determined from the i vs S diagram of state of the steam, knowing the vapor pressure of the "hot" steam at the point of interest. Since the pressure of the "hot" stream increases with an increase in μ, the temperature of the "hot" stream will be increased accordingly.

When the vortex tube is supplied with superheated vapor the "cold" and "hot" streams contain superheated vapor. The temperature level of the supplied superheated vapor at which the vapor of the "cold" stream is also superheated is determined by the degree of expansion of the vapor in the tube, the weight fraction μ of the cold stream, the pressure of the supplied vapor, and the operating efficiency of the tube, for which it is necessary to select the optimum geometrical proportions of the vortex tube.

It was established experimentally that when the vortex tube operates on superheated steam with a pressure of up to $5.9 \cdot 10^5$ N/m^2 the optimum diffuser angle of the hot end is 3° with a length of not less than 15 tube diameters. It is recommended to select the diameter of the diaphragm opening from the expression $d/D = 0.43 + 23\mu$. The nozzle area is selected as a function of the operating conditions of the vortex tube.

Dep. 1765-75, May 8, 1975.
Khar'kov Aviation Institute.
Original article submitted August 12, 1971.

WAVEFRONT KINEMATICS AND THE HYDRODYNAMIC DESCRIPTION OF ACOUSTIC DISPERSION IN HETEROGENEOUS MEDIA

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UDC 534.222

Equations obtained from the equations of hydrodynamics are used for the description of acoustic dispersion in liquids. The way of obtaining the dispersion equations using microscopic concepts is well known [1]. The question of the origin of the universalism of the dispersion equations is taken up in the report. The general principles controlling the behavior of a wavefront in a medium are revealed within the framework of the laws of the geometry of four dimensions – of kinematics [2]. The dispersion equation for the absorption coefficient is obtained from the representation of the wavefront $f(x, y, z, t)$ in the form of the product of two functions of the amplitude $A(x, y, z, t)$ and the phase $F(\omega t - k \cdot r)$ using only geometrical theorems. In a comparison with the dispersion equations obtained from the equations of hydrodynamics those physical propositions which are adopted in the transition from kinematics to hydrodynamics are established. The relationship of the kinematic and hydrodynamic approaches to the description of dispersion is analyzed on the example of experimental studies performed by the authors on acoustic dispersion in a heterogeneous medium consisting of rosin particles in an aqueous solution of ethyl alcohol. Disagreement with calculations based on the Lamb and Einstein equations [3] is discovered. A calculation by the dispersion equation of Predvoditelev [4] gives agreement with measurements of the absorption coefficient if one allows for the particle form factor calculated from data of viscosimetric measurements.

It is noted that the kinematic analysis of acoustic dispersion opens up wider possibilities for the establishment of general relationships in the propagation of waves in liquids and gases.

LITERATURE CITED

1. N. P. Kasterin, Wave Propagation in a Heterogeneous Medium [in Russian], Moscow (1903).
2. A. S. Predvoditelev and A. A. Solov'ev, A New Look at Problems of Physical Acoustics [in Russian], Izd. Mosk. Gos. Univ. (1974).
3. A. Einstein, Collection of Scientific Works [Russian translation], Vol. 3, Nauka, Moscow (1966), p. 75.
4. A. A. Solov'ev, Inzh.-Fiz. Zh., 20, No. 6 (1971).

Dep. 1760-75, May 11, 1975.
Original article submitted June 7, 1973.

CONVECTIVE HEATING IN THE PRESENCE OF TIME-VARYING HEAT-EXCHANGE COEFFICIENTS

P. V. Tsoi

UDC 536.21

In the report an analysis is made of an approximate analytical method of solving the boundary problem of nonsteady heat conduction in three bodies of classical shape with a variable heat-exchange coefficient:

$$\left\{\frac{\partial T}{\partial \xi} = \mathrm{Bi}\,(\mathrm{Fo})\,[\varphi\,(\mathrm{Fo}) - T\,(\xi,\ \mathrm{Fo})]\right\}_{\xi=1}. \tag{1}$$

The temperature distribution in a plate, a cylinder, and a sphere is determined in a family of functions of the type

$$T\,(\xi,\ \mathrm{Fo}) = \varphi\,(\mathrm{Fo}) + a_1\,(\mathrm{Fo})\left[\frac{\mathrm{Bi}\,(\mathrm{Fo})+2}{\mathrm{Bi}\,(\mathrm{Fo})} - \xi^2\right], \tag{2}$$

which satisfy the boundary condition (1) for any bounded functions $a_1(\mathrm{Fo})$. The indeterminate function $a_1(\mathrm{Fo})$ is found by the method of orthogonal projection of the discrepancy of the heat-conduction equation. An explicit solution is presented for a cylinder with $\mathrm{Bi}(\mathrm{Fo}) = \mathrm{Bi}_0(1 + \mathrm{Pd}\cdot\mathrm{Fo})$ and $\mathrm{Pd} = \beta$:

$$\theta\,(\xi,\ \mathrm{Fo}) = 1 - (1-\theta_0)\,\frac{2\,\mathrm{Bi}_0\,(1+\beta\,\mathrm{Fo})}{\mathrm{Bi}_0+4}\left[\frac{\mathrm{Bi}_0\,(1+\beta\,\mathrm{Fo})+2}{\mathrm{Bi}_0\,(1+\beta\,\mathrm{Fo})} - \xi^2\right]\times$$
$$\times\left[\frac{\mathrm{Bi}_0^2\,(1+\beta\,\mathrm{Fo})^2 + 6\,\mathrm{Bi}_0\,(1+\beta\mathrm{Fo}) + 12}{\mathrm{Bi}_0^2 + 6\,\mathrm{Bi}_0 + 12}\right]^{\left(\frac{6}{\beta\mathrm{Bi}_0} - \frac{1}{2}\right)}\times$$
$$\times\exp\,(-6\,\mathrm{Fo})\cdot\exp\left\{\frac{36}{\sqrt{3}\,\beta\,\mathrm{Bi}_0}\left[\mathrm{arctg}\,\frac{\mathrm{Bi}_0\,(1+\beta\,\mathrm{Fo})+3}{\sqrt{3}} - \mathrm{arctg}\,\frac{\mathrm{Bi}_0+3}{\sqrt{3}}\right]\right\}. \tag{3}$$

The results of a calculation of the temperature at the surface and the axis of the cylinder and a comparison with the data of other authors are presented in Table 1. From (3) as $\beta \to 0$ we obtain

$$\theta^*\,(\xi,\ \mathrm{Fo}) = \frac{T\,(\xi,\ \mathrm{Fo}) - T_0}{T_m - T_0} = 1 - \frac{2\,\mathrm{Bi}_0}{\mathrm{Bi}_0+4}\left(\frac{\mathrm{Bi}_0+2}{\mathrm{Bi}_0} - \xi^2\right)\exp\,[-A\,(\mathrm{Bi}_0)\,\mathrm{Fo}], \tag{4}$$

where

$$A\,(\mathrm{Bi}_0) = \frac{6\,\mathrm{Bi}_0\,(\mathrm{Bi}_0+4)}{\mathrm{Bi}_0^2 + 6\,\mathrm{Bi}_0 + 12}. \tag{5}$$

The solution (4) gives better agreement with exact solutions for $\mathrm{Fo} \geq 0.05$, while the expression for $A(\mathrm{Bi}_0)$ only slightly exceeds the square of the first root of the equation (μ_1^2):

$$\frac{J_0\,(\mu)}{J_1\,(\mu)} = \frac{\mu}{\mathrm{Bi}_0}.$$

TABLE 1. (1)

Fo	Surface θ (1, Fo): from nomogram of [2]	Surface θ (1, Fo): from solution of [1]	Surface θ (1, Fo): from Eq. [3]	Center θ (0, Fo): from nomogram of [2]	Center θ (0, Fo): from solution of [1]	Center θ (0, Fo): from Eq. [3]
0,1	0,57	0,620	0,566	0,230	0,220	0,221
0,2	0,66	0,710	0,663	0,34	0,360	0,349
0,4	0,80	0,820	0,778	0,57	0,580	0,572
0,6	0,86	0,890	0,869	0,75	0,740	0,753
0,8	0,92	0,940	0,915	0,82	0,850	0,834
1	0,95	0,960	0,955	0,92	0,890	0,910
2	1,0	0,998	1,000	1,00	0,993	0,996
3	1,0	0,999	1,000	1,00	0,998	1,000

A simple and sufficiently exact theoretical method of studying thermal stresses in heat-releasing elements in the period of elastic deformations produced by temperature gradients, when the laws of distribution and time variation of the local internal heat sources are given, is presented in the second part of the article.

LITERATURE CITED

1. Yu. S. Postol'nik, Izv. Vyssh. Uchebn. Zaved., Chern. Metallurg., No. 6 (1970).
2. V. V. Salomatov and É. I. Goncharov, Izv. Akad. Nauk SSSR, Énerget. Transport, No. 6 (1968).

Dep. 1772-75, April 1, 1975.
Original article submitted April 24, 1974.

SOME PROBLEMS OF HEAT CONDUCTION WITH A TRANSFER COEFFICIENT WHICH DEPENDS ON THE COORDINATES

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UDC 536.21

The solution of the heat-conduction equation with variable coefficients is a matter of great practical and theoretical importance. Up to the present time, as noted in [1, 2, 3], exact analytical solutions have been obtained only for a very limited class of problems.

In the present paper we investigate the heat-conduction equation with variable coefficients

$$\operatorname{div}(\lambda \operatorname{grad} T) = c\gamma \frac{\partial T}{\partial t} - W(x, y, z, t).$$

Here $T = T(x, y, z, t)$, $\lambda = \lambda(x, y, z)$, $c = c(x, y, z)$, $\gamma = \gamma(x, y, z)$ are transformed into an equation with constant coëfficients without changing the domain of integration only by using the transformation

$$T(x, y, z, t) = \frac{\exp \frac{1}{2}(c_1 x + c_2 y + c_3 z + c_4)}{\sqrt{\lambda(x, y, z)}}\, \theta(x, y, z, t), \tag{1}$$

where $c_j (j = 1, 2, 3, 4)$ are arbitrary constants, subject to the condition that

$$c\gamma/\lambda = \text{const}, \quad \lambda(x, y, z) = \lambda_1(x)\lambda_2(y)\lambda_3(z),$$

and $\lambda_i(\eta)$ $(i = 1, 2, 3;\ \eta = x, y, z)$ takes the form of one of the expressions

$$\begin{aligned} \lambda_i(\eta) &= (a_i\eta + b_i)^2, \\ \lambda_i(\eta) &= [A_i \cos(k_i\eta + a_i) + B_i \sin(k_i\eta + b_i)]^2, \\ \lambda_i(\eta) &= [A_i \operatorname{ch}(k_i\eta + a_i) + B_i \operatorname{sh}(k_i\eta + b_i)]^2, \end{aligned} \tag{2}$$

A_i, B_i, k_i, a_i, b_i are arbitrary constants.

We consider the problem of nonstationary heat conduction for an unbounded plate of thickness l assuming that the coefficient of thermal conductivity of the material varies according to the law (2), that at the initial instant of time the plate has a temperature $T = \psi(x)$, and that for $t > 0$ the surfaces of the plate have, respectively, the temperatures $T = T_1 + \varphi_1(t)$ and $T = T_2 + \varphi_2(t)$, where T_1, T_2 = const.

In the solution of the problem we make use of special cases of the transformations (1) [4] with subsequent application of the classical Fourier method.

The solution is found in the form of an infinite series.

From the resulting solution for A = 1, B = − 1, a = b = 0, $\varphi_1 = \varphi_2 = 0$, $T_2 = T_C$, $T_1 = 0$ in the stationary case ($t \to \infty$) we obtain the solution proposed in [1].

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Moscow (1967).
2. V. G. Korenev, Some Problems in the Theory of Elasticity and Heat Conduction Which Are Solvable in Terms of Bessel Functions [in Russian], Fizmatgiz, Moscow (1960).
3. A. F. Chudnovskii, Thermophysical Characteristics of Dispersed Materials [in Russian], Fizmatgiz, Moscow (1962).
4. V. S. Egorov, "Three-dimensional nonstationary problem of heat conduction for a sector of a hollow cylinder of finite dimensions," in: Mathematical Physics [in Russian], Izd. Akad. Nauk UkrSSR, No. 9 (1970).

Dep. 1775-75, February 28, 1975.
Original article submitted July 17, 1974.

SUFFICIENT CONDITIONS FOR THE APPLICATION OF THE GOODIER METHOD

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UDC 536.2.01

A combination of the methods of Grinberg–Koshlyakov and Goodier turns out to be effective in solving thermoelasticity problems in those cases when the temperature is a solution of some linear heat-conduction problem. The temperature and thermoelastic potential are hence obtained in the form of series and the stresses are expressed in terms of the partial derivatives of the thermoelastic potential. However, in the absence of uniform convergence of the temperature series, doubt is cast on the possibility of using the Goodier method which is associated with term-by-term differentiation. The nature of the convergence of the series mentioned can be established in each specific case. For instance, the temperature field of a hollow cylinder of length l with heat sources of constant intensity q distributed over the volume within whose cavity a fluid at zero temperature flows can be represented as the following double series:

$$t = \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \theta_{np} \sin \frac{(2p+1)\pi z}{l} U_0\left(\mu_n \frac{r}{R_1}\right).$$

If the initial cylinder temperature is zero, then the coefficients Θ_{np} are such that the series converges absolutely and uniformly. Differentiation of the thermoelastic potential which is expressed in a form analogous in structure is completely admissible in this case. Moreover, the double series, which yield an expression for the stresses, also converge absolutely and uniformly. In addition to compliance with the sufficient conditions for the application of the Goodier method, there is still a practically convenient consequence consisting in rapid convergence of the stress series.

If the initial cylinder temperature is not zero, then the series mentioned converges nonuniformly. However, this case can be reduced to the preceding one if the exact initial condition is replaced by the approximate condition

$$f = \begin{cases} -T\dfrac{z}{\Delta}, & 0 \leqslant z < \Delta, \\ -T, & \Delta \leqslant z < l - \Delta, \\ T(z-l)/\Delta, & l - \Delta \leqslant z \leqslant l. \end{cases}$$

Here the interval Δ is selected sufficiently small.

Again using the Goodier method, we obtain the stress in the form of a double series whose coefficients do not decrease more slowly than $1/(2p+1)^2\mu_n^2$.

Dep. 1770-75, May 7, 1975.
Original article submitted July 24, 1974.

FIELD OF TEMPERATURE STRESSES IN THE WALL OF THE SCREEN PIPE OF A STEAM GENERATOR FOR A PERIODIC CHANGE IN THE HEAT-EXCHANGE COEFFICIENT

V. F. Stepanchuk and M. L. Guris

UDC 621.1.016.4:536.24

Operating conditions for the metal of screen pipes of high-pressure steam generators are considered at high heat fluxes when transient boiling, characterized by a periodic change in the heat-exchange coefficient, is established.

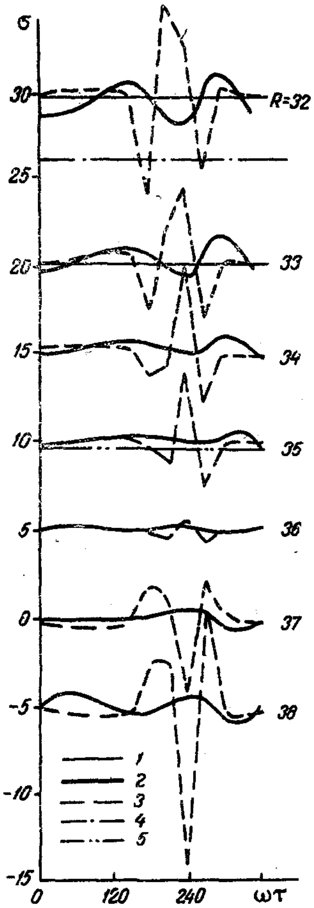

Fig. 1. Stresses in the wall of a screen pipe: 1) stationary mode; 2) $\omega = 1$ rad/sec; 3) $\omega = 0.1$ rad/sec; 4) yield point; 5) allowed stress rad/sec. σ, kg/mm^2; $\omega\tau$, deg.

The computation is carried out by means of the classical equations of thermal stresses in the pipe wall [1]. Results of determining the temperature field of a flat wall with a periodically varying heat-exchange coefficient, presented in [2], are used in the computation.

The temperature stresses are computed on a "Minsk-22" electronic digital computer. The results are represented in the figure, from which it follows that the stresses on the inner surface of screen pipe in high-pressure steam generators with heat fluxes on the order of 580 kW/m^2 will exceed the yield point even for stationary boiling, which is nevertheless allowed by the adaptability conditions. In the case of spoilage of the boiling mode, temperature waves are propagated in the pipe wall, in whose presence exceeding the yield point results in fatigue rupture of the metal.

Because of the prolonged effect of transient boiling, cracks and pits appear on the pipe inner surface, where fracture of the screen surfaces in the zone of maximum heat fluxes of the TM-84 of the Polotskii TÉTs-2, whose parameters were the basis of the computation presented, can be an example of such damage.

LITERATURE CITED

1. S. D. Ponomarev et al., Strength Analyses in Machine Construction [in Russian], Vol. 2, Moscow (1958).
2. V. F. Stepanchuk and M. L. Guris, Inzh. Fiz. Zh., 23, No. 5 (1972).

Dep. 1763-75, April 28, 1975.
Original article submitted February 14, 1975.

ANALYSIS OF SOLIDIFICATION IN A FLAT, SPHERICAL, AND CYLINDRICAL LAYER

M. I. Dubovis

UDC 536.24.02

The temperature field in a solidifying layer is described by a one-dimensional, homogeneous heat-conduction equation. The temperature is zero on the moving boundary (solidification front).

Latent heat and heat of superheat are eliminated from the liquid phase through the front. It is assumed as an approximation that the heat content of the liquid phase during solidification is a known function of the front coordinates and of the layer boundary. This results in the intensity of superheat elimination being, moreover, linearly dependent on the velocity of front motion. Two problems are examined: solidification on the outside and from within. The problems are reduced to dimensionless form. The known expressions for the temperature given by B. T. Borisov, B. Ya. Lyubov, D. E. Temkin, E. Ya. Iodko, and M. I. Dubovis are used. The expression for the temperature is substituted into the condition given on a fixed boundary. The dimensionless coordinate of the front is replaced by the Maclaurin series

$$Y = \sum_{n=1}^{\infty} \frac{Y^{(n)}(0)}{n!} F^n,$$

where F is the Fourier criterion. By using successive differentiation and passage to the limit as F, Y → 0, the following quantities are evaluated: $Y'(0)$, $Y''(0)$, ... Formulas are presented to evaluate $Y^{(n)}(0)$, n = 1, 2, 3, for cases when conditions of the first, second, and third kind with an inhomogeneity, an arbitrary function of the time, are given on the fixed boundary. These formulas contain derivatives of this function of time, the derivatives $Y^{(n)}(0)$ of previous numbers, and expressions related to the heat of superheat. Examples of numerical computations are presented. Expressions for the inverse problem, the determination of conditions on the fixed boundary for a given law of consumption of the heat of superheat when the front motion will depend linearly or on the square root of the time, are given for a plane and spherical layer.

Dep. 1767-75, November 11, 1975.
Original article submitted November 21, 1973.

NONLINEAR HEAT-CONDUCTION PROBLEM FOR THE COMPUTATION OF TWO-LAYERED STRUCTURES SUBJECTED TO FIRE

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UDC 536.212:614.841.34

The need often arises in structural thermophysics for an analysis of the fire resistance of reinforced concrete structures subjected to fire. The solution of a heat-conduction problem is a component part of such an analysis.

An approximate analytic solution of this problem taking into account the change in the thermophysical characteristics of the material with temperature is presented in this paper in an example of a two-layered structure.

The boundary condition on the heated surface is taken on the basis of numerous fire-resistance tests of reinforced concrete structures [1]. The thermal contact between the layers is considered ideal. The method of the small parameter, for which a dimensionless coefficient characterizing the change in heat conduction of the material with the increase in temperature is taken, is used to solve the problem. In conformity with this method, the solution of the problem is represented as the sum of series with terms containing the small parameter in increasing powers. Substituting the series in the differential equation describing the process of heating of the structure, differentiating term-by-term, and then equating terms in identical powers of the small parameter, we consequently obtain a system of linear differential equations to determine the members of the series, i.e., to find the zero, first, second, and subsequent approximations.

The system for the zero approximation is the mathematical formulation of the problem under consideration, but under the condition of independence of the thermophysical characteristics of the material from the temperature. This system is solved by an operational method. Subsequent systems for the first, second, etc. approximation are solved by using the finite Fourier integral cosine transform. Expressions are finally presented for the change in temperature in two- and single-layer structures subjected to fire.

The results of the computations are compared with experimental data. It is shown that the second approximation permits obtaining convergence within 1-3% limits.

LITERATURE CITED

1. V. P. Bushev, V. A. Pchelintsev, V. S. Fedorenko, and A. I. Yakovlev, Fire Resistance of Buildings [in Russian], Stroiizdat (1970).

Dep. 1759-75, May 19, 1975.
Original article submitted April 1, 1974.

CALCULATION OF POTENTIAL FIELDS BY USING EQUIVALENT INTEGRAL PARAMETERS

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UDC 536.24

To study heat conduction we must calculate potential fields in regions of complex form that do not allow us to directly use existing analytic methods.

We can simplify the problem considerably if we study a field only in its separate parts instead of in the entire region. We assume the method of equivalent parameters in the study for this class of problems. The essence of the method is the division of the initial region into more simple subregions: the basic subregion in which we study the field according to the conditions of the problem, and the auxiliary subregion. To determine the field in the basic subregion we consider the effect of the auxiliary subregion by introducing its equivalent integral parameters: the temperature change and the heat resistance on the surface where the subregions adjoin.

The indicated equivalent parameters are determined in proceeding from a known electrothermal analogy by means of solving simple boundary-value problems in the auxiliary subregion.

This method can be used to divide the initial region not only into two, but into a larger number of subregions.

A test problem with an exact analytic solution is examined in the study to estimate the accuracy of the method. By comparing it with the solution used in the equivalent parameter method, we can show the reliability of the latter.

As an example of the practical application of the method we present the solution of problems for the temperature distribution in a semibounded body with a cylindrical ledge when the body heat is realized through the end of this ledge, and the conditions of convective heat exchange with the environment are satisfied on the remaining sections of the boundary surface.

Dep. 1769-75, May 8, 1975.
Original article submitted October 28, 1974.

CALCULATION OF HEAT FLUX ALONG DUCTS PLACED IN CONSOLIDATED GAS

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UDC 536.21

We study a problem for determining heat flux along ducts in cryostats with consolidated gas in order to estimate the superheating of the cryostat objects with respect to the temperature of a cooling agent.

We obtain a differential equation of thermal conductivity for the duct

$$q_R(x, \tau) = \frac{2\lambda_{eq}[T(x, \tau) - T_0]}{d_1 \ln \frac{D(x, \tau)}{d_1}}, \tag{1}$$

$$C\rho \frac{\partial T(x, \tau)}{\partial \tau} = \lambda \frac{\partial^2 T(x, \tau)}{\partial x^2} - \frac{4q_R(x, \tau)}{d_1}, \tag{2}$$

$$D^2(x, \tau) = d_1^2 + \frac{4d_1}{\rho_0 r_0} \int_0^{\tau} q_R(x, \tau)\, d\tau \tag{3}$$

with the boundary conditions

$$T(x, 0) = T_0, \quad \frac{\partial T(0, \tau)}{\partial x} = \frac{T_c - T(0, \tau)}{L}; \quad T(l, \tau) = T_0. \tag{4}$$

We can solve system of equations (1)-(4) by a numerical difference method.

We present equations of the computer calculations for the temperature variation along the duct length for the external diameter of the steam layer around the duct, and for the superheating of different cryostat elements with respect to the cooling agent, owing to the heat flux along the ducts.

NOTATION

q_r, specific heat flux across the duct surface; C, ρ, λ, heat capacity, density, and thermal conductivity of the duct material; r_0, ρ_0, sublimation heat and density of consolidated gas; λ_{eq}, equivalent coefficient of the thermal conductivity of the steam layer of the sublimating cooling agent; D, external diameter of the steam layer around the duct; L, duct length between the cryostat cover and consolidated cooling agent; l, length of the duct in the cooling agent; T_0, temperature of the cooling agent; T_c, temperature of cryostat cover.

Dep. 1216-75, March 17, 1975.
Original article submitted August 24, 1974.

SOLUTION OF A PROBLEM OF HEAT AND MOISTURE EXCHANGE

M. N. Shafeev

We study a dimensionless system of differential equations of heat and moisture exchange that fully describe the basic characteristics in the actual process of the freezing of rocks around shallow holes.

In soil, as in any multicomponent medium, the temperature of the phase transition considerably depends on the water quantity and varies at wide limits thanks to various forms of the relation between the moisture and the mineral particles (soil frame). Thus we introduce the dimensionless freezing point for the basic moisture mass in the soil.

We obtain generalized solutions of the indicated system of equations in finite form by the method suggested for the defined boundary conditions. These analytic solutions describe the character of the distribution of the dimensionless potentials for heat and moisture exchange in the freezing zones of the soil around a shallow hole, along with the time and velocity of soil freezing; the variation of the initial moisture and the dynamics of the moisture-transport change are taken into account on the freezing surface S_1. Based on these results, we can establish the capacity of the moisture migration to the surface S_1 from the cooling zone, the removal of heat from the freezing soil, the refrigerating capacity, and other regime parameters of the freezing soil.

In the study we use a method for realizing the obtained solutions in calculations of thermal engineering. In graphs we illustrate the results of the numerical solutions for determining the dynamics of the distribution of the heat- and moisture-transfer potentials in the freezing soil around a shallow hole, along with the time and velocity of freezing and the dynamics of the moisture-transport change on the surface S_1.

An analysis of the solutions obtained shows that all the solutions have a simple structure and contain rapidly converging series. Repetition of the identical operations in the numerical calculations is convenient for realizing these solutions on a computer.

Our studies show that with the increasing effect of the phase transition on the moisture transfer (in terms of a modified Kossovich criterion) and with a decreasing Lykov criterion, the development of the heat-transfer potentials in the freezing soil considerably advances the development of the moisture-transfer potentials, and thus the velocity of the freezing soil increases. In the freezing soil zone, with the exception of water-saturated grains, we observe the maximum dimensionless moisture-transfer potential, in which there is a shift to an increasing Fourier criterion. This maximum quantity increases with an increasing coefficient of the thermal gradient; its axis shifts towards the heat flow, and the moisture transfer intensifies in the freezing soil. The moisture-transport change decreases with an increasing Fedorov criterion on the surface S_1; the maximum dimensionless potential of moisture transfer shifts towards the motion of this surface and is consequently reduced.

Under the physical assumptions in the study, we also apply the data of the solutions to the melting process by a simple replacement of the indices characterizing the various states.

Dep. 1219-75, July 1, 1975.
Original article submitted January 8, 1974.